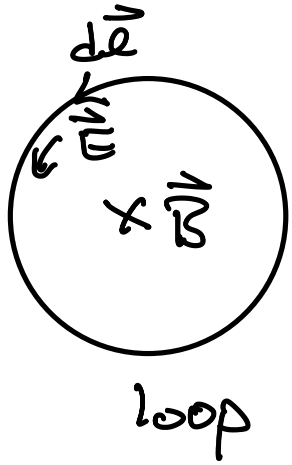
Electromagnetic waves

Faraday's law: $EMF = -\frac{d\Phi_m}{dt}$

$EMF = \oint_{loop} \vec{E} \cdot d\vec{\ell}$



$\Phi_m = \int_{\text{area of loop}} \vec{B} \cdot d\vec{A}$

so $\oint_{loop} \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int_{\text{area of loop}} \vec{B} \cdot d\vec{A}$

relates changing $\vec{B}$ to an $\vec{E}$

Ampere's law: $\oint_{loop} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{in} + \varepsilon_0 \frac{d\Phi_E}{dt}$

$\varepsilon_0 \frac{d}{dt} \int_{\text{area of loop}} \vec{E} \cdot d\vec{A}$

relates changing $\vec{E}$ to a $\vec{B}$

so $\frac{d\vec{B}}{dt} \to \vec{E}$ and $\frac{d\vec{E}}{dt} \to \vec{B}$

this is why there is Electromagnetic radiation (EM radiation)

and velocity $v = c = 3\times10^8$ m/s = 0.3m * 1 GHz

various wavelength/frequencies have names

λ (m)		
10^{-14}	Gamma rays	
10^{-12} - 10^{-10}	X-rays	
10^{-8}	UV	10 nm
10^{-7}	visible	$\sim \frac{1}{3}$ - $\frac{1}{2}$ μm
10^{-6} - 10^{-4}	IR	μm
10^{-2}	radar	mm
1	FM	m
10	TV	10 m
100	short wave	100 m
10^4	AM	10^4 m

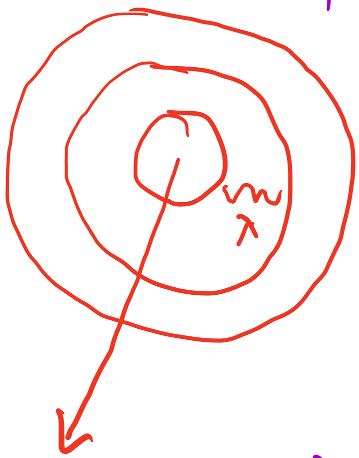
<u>But</u> in this chapter we focus on visible

$\lambda \ll$ dimensions of normal things

$\Rightarrow$ wave like nature hard to see

so can treat EM radiation as rays

EM waves propagate in 3 dimensions



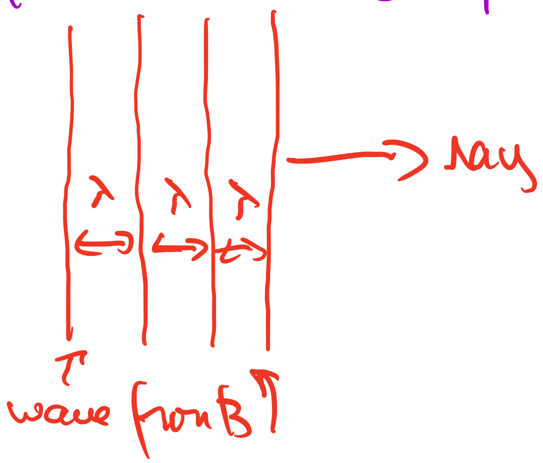
→ wave fronts
usually at same phase
dist between fronts
usually the wavelength

ray = direction of wave front at that point

for EM radiation "infinitely far" from the source:

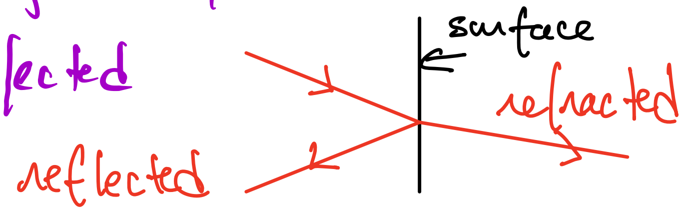
$r >> \lambda$

wave front ⊥ ray - plane waves

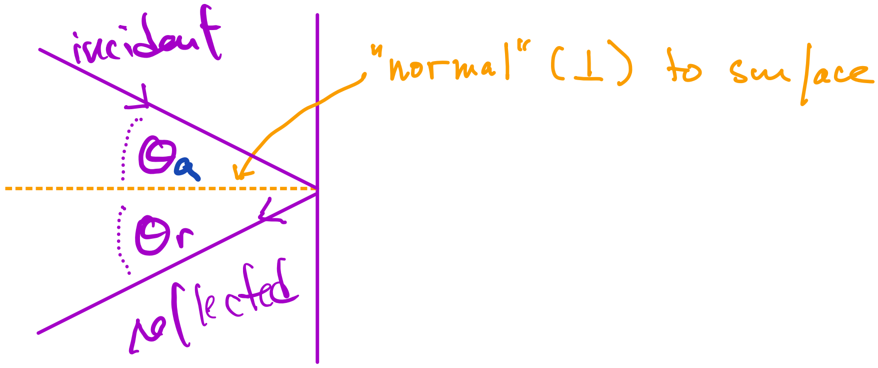


light waves hitting a surface can be <u>both</u>

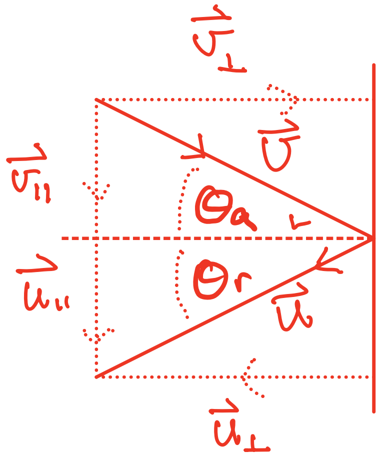
transmitted and reflected



Reflection at a surface



specular reflection – from smooth surface like mirror
– surface has no unusual structure



$\vec{v}$ = incident light
$\vec{u}$ = reflected "

$\vec{v} = \vec{v}_{\parallel} + \vec{v}_\perp$ ($\vec{v}_{\parallel}$ parallel to surface; $\vec{v}_\perp$ ⊥ to surface)

then $\tan\theta_a = v_{\parallel} / v_\perp$

reflected light: $\vec{u}_{\parallel} = \vec{v}_{\parallel}$ so ∥ unchanged
$\vec{u}_\perp = -\vec{v}_\perp$ so ⊥ reverses dir

so $\vec{u}_{\parallel} + \vec{u}_\perp = \vec{v}_{\parallel} - \vec{v}_\perp$

reflected angle $\tan\theta_r = \dfrac{u_{\parallel}}{u_\perp} = \dfrac{v_{\parallel}}{v_\perp} = \tan\theta_a$

⇒ law of reflection: $\theta_r = \theta_a$
(not for diffuse reflection! - need very smooth surface)

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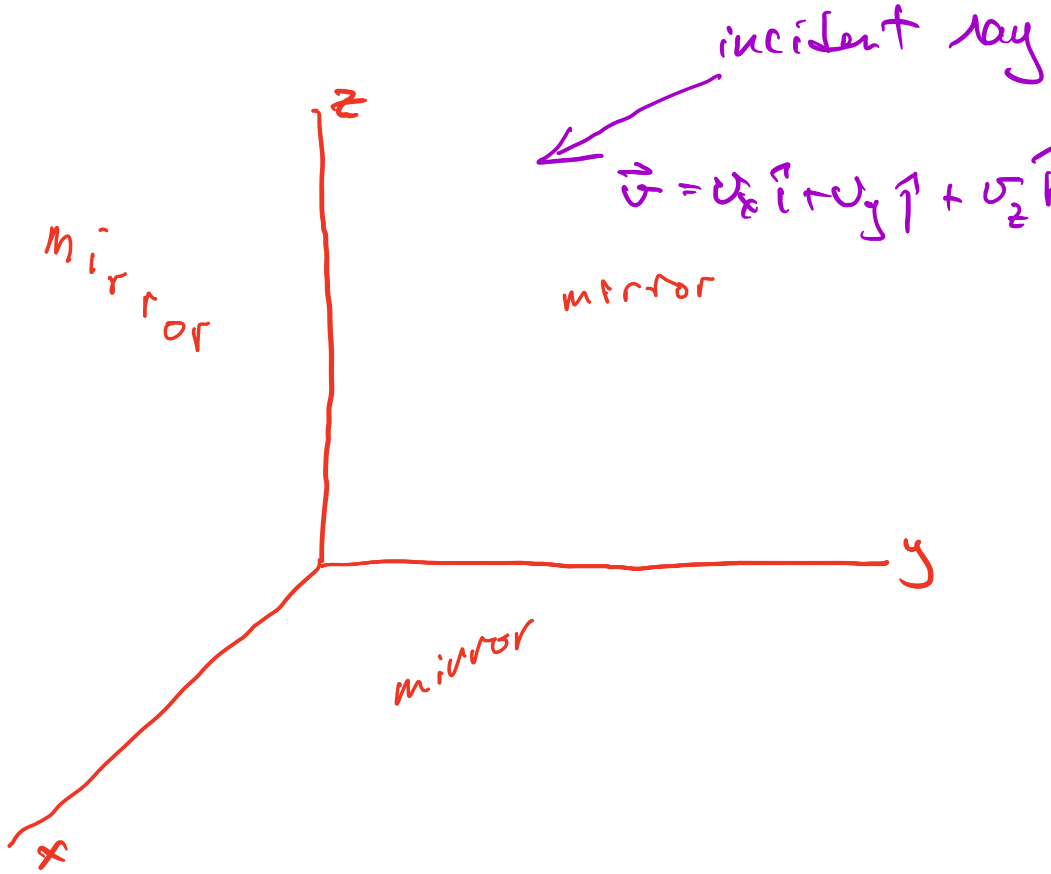
Reflection at mirrors is almost perfect
Regular mirror, loss is few % (absorbed)
Can make highly reflective mirrors w/ special coating, loss < 0.001% or better

Corner reflector - 3 mirrors that makes a corner



incident wave reflects off of all 4 mirrors

1^{st} reflection // xz plane - y component reverses
2^{nd} " " xy " - z " "
3^{rd} " " yz " - x " "

$\vec{v}_f = -v_x \hat{i} - v_y \hat{j} - v_z \hat{k} = -\vec{v}$!

corner reflector will send beam back in same direction originated?

Refraction

speed of light is $c = 3 \times 10^8$ m/s in vacuum

in materials, light slows down
→ interaction w/ atoms & molecules

$v_n = \frac{c}{n}$ n = index of refraction, v_n = vel in material

	vacuum	glass	water	diamond	zircon
n:	1	~1.5	~1.3	~2.5	~2.0

note: in materials, freq f does not change!
$c = \lambda f$ and $v_n = \lambda_n f = \frac{c}{n} \lambda f$

$\therefore \lambda_n = \frac{\lambda}{n}$ λ is in vacuum

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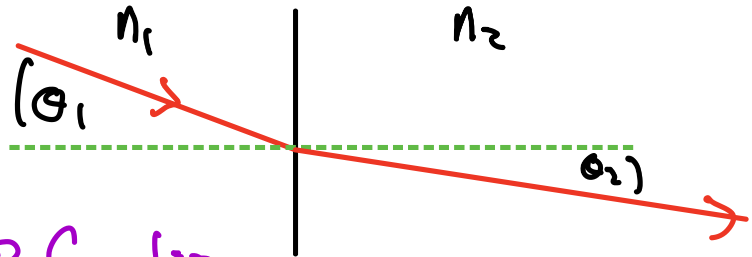
Refraction

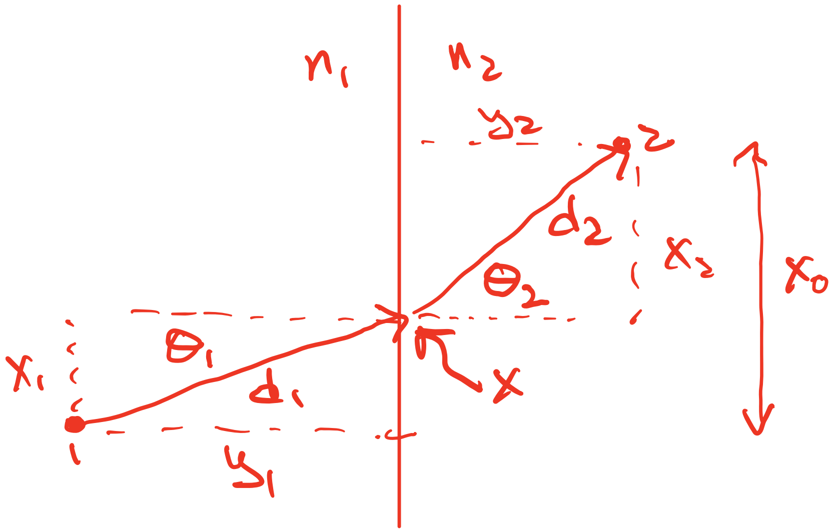
- refraction is the bending of light when transitioning between regions w/ different index of refractions



Law of Refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

derivation using Principle of Least Time



- x is the variable that tells you where you hit the boundary
- x_1, y_1, x_2, y_2 are fixed
- v_1 and v_2 are velocities in the 2 regions

$\Rightarrow$ calculate time going from point 1 to point 2 as a function of "x"

$t = t_1 + t_2 \qquad t_1 = d_1/v_1 \qquad t_2 = d_2/v_2$

⇒ we can vary x to find value that minimizes total time $t_1 + t_2$. note: $x_1 + x_2 = x_0$ is constant

y_1 & y_2 are constant

$x = x_1$

$$d_1 = \sqrt{x_1^2 + y_1^2} = \sqrt{x^2 + y_1^2} = v_1 t_1$$

$$d_2 = \sqrt{x_2^2 + y_2^2} = \sqrt{(x_0 - x)^2 + y_2^2} = v_2 t_2$$

so $$t = t_1 + t_2 = \frac{\sqrt{x^2 + y_1^2}}{v_1} + \frac{\sqrt{(x_0 - x)^2 + y_2^2}}{v_2}$$

to find minimum take $\frac{dt}{dx} = 0$

$$\frac{dt}{dx} = \frac{1}{2} \frac{2x}{\sqrt{x^2 + y_1^2}\, v_1} + \frac{1}{2} \frac{2(x_0 - x)(-1)}{v_2 \sqrt{(x_0 - x)^2 + y_2^2}} = 0$$

or $$\frac{x}{v_1 \sqrt{x^2 + y_1^2}} = \frac{(x_0 - x)}{v_2 \sqrt{(x_0 - x)^2 + y_2^2}}$$

note: $\frac{x_1}{\sqrt{x^2 + y_1^2}} = \sin\theta_1$ and $\frac{(x - x_0)}{\sqrt{(x_0 - x)^2 + y_2^2}} = \sin\theta_2$

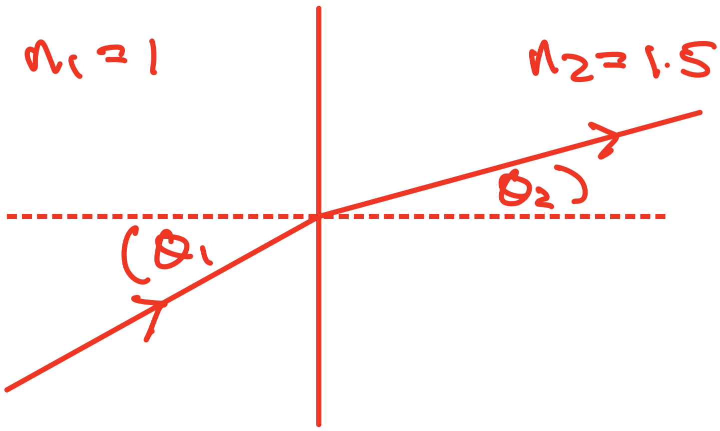
substitute $v_1 = \frac{c}{n_1}$ and $v_2 = \frac{c}{n_2}$

so $$\frac{\sin\theta_1}{c/n_1} = \frac{\sin\theta_2}{c/n_2}$$

gives $n_1 \sin\theta_1 = n_2 \sin\theta_2$

cases: $n_1 < n_2 \Rightarrow \sin\theta_1 > \sin\theta_2$

eg. air → glass $\therefore \theta_1 > \theta_2$ so is bent <u>towards</u> normal



for $n_2 < n_1$, its opposite → as if light goes backwards in time

note: there will always be some reflection at the boundary

for glass ~ loss is around 4% intensity

<u>Rule</u>

so $n_2 > n_1$ (from smaller to larger n)
$\Rightarrow$ bends towards normal

for $n_2 < n_1$ (from larger to smaller n)

$\Rightarrow$ bends away from normal

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note: $c = \lambda f$ for light

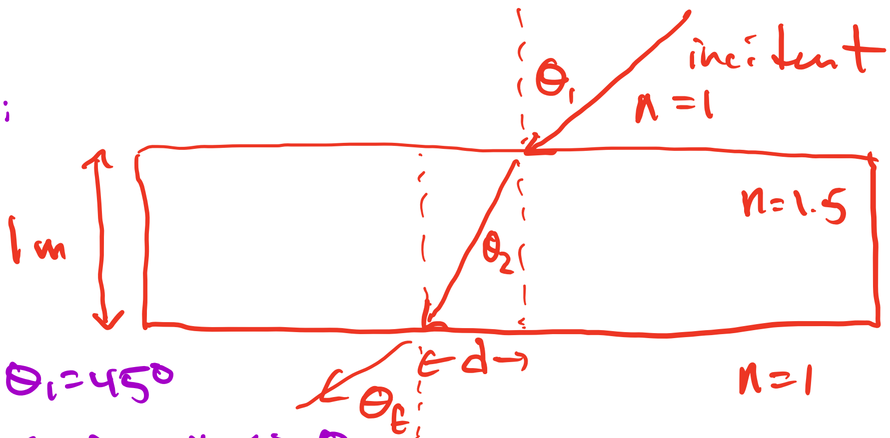
f = vibration freq, λ = how far in the medium for 1 cycle

so if $v = \frac{c}{n} = \frac{\lambda f}{n}$ then $\lambda_n = \frac{\lambda}{n}$ wave length in medium

but f is the same

ex: light goes thru slab of glass

ex:



let $\theta_1 = 45°$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 \sin\theta_1 = \sin 45° = \frac{1}{\sqrt{2}} = n_2 \sin\theta_2 = 1.5 \sin\theta_2$

$\theta_2 = \sin^{-1}\left(\frac{1}{1.5\sqrt{2}}\right) = \sin^{-1}(0.47) = 28.1°$ ⇒ bent towards normal

$\tan\theta_2 = \frac{d}{\text{thickness}} = \frac{d}{1\text{m}}$

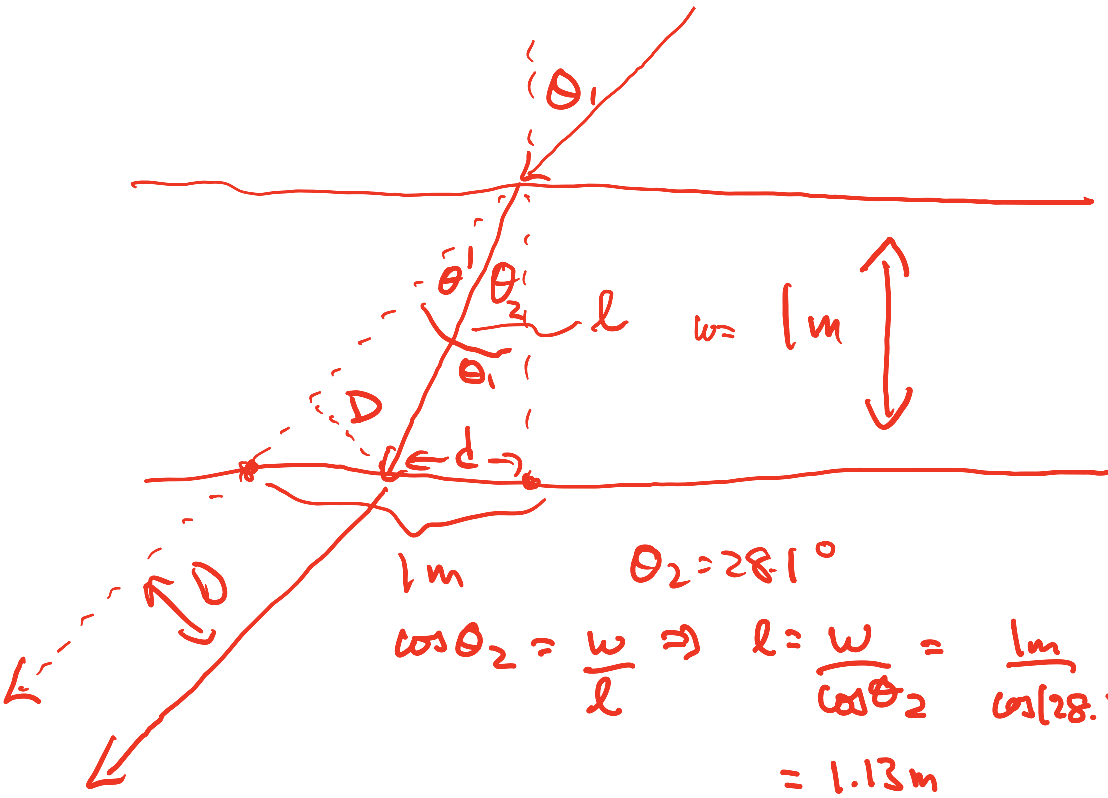
$d = 1 \cdot \tan\theta_2 = \tan(28.1°) = 0.53\text{m}$

what angle does light leave slab?

$n_2 \sin\theta_2 = n_1 \sin\theta_f$ ⇒ same eqn as when it entered

so $\theta_f = 45°$

what is displacement D when it exits?



$\theta_2 = 28.1°$

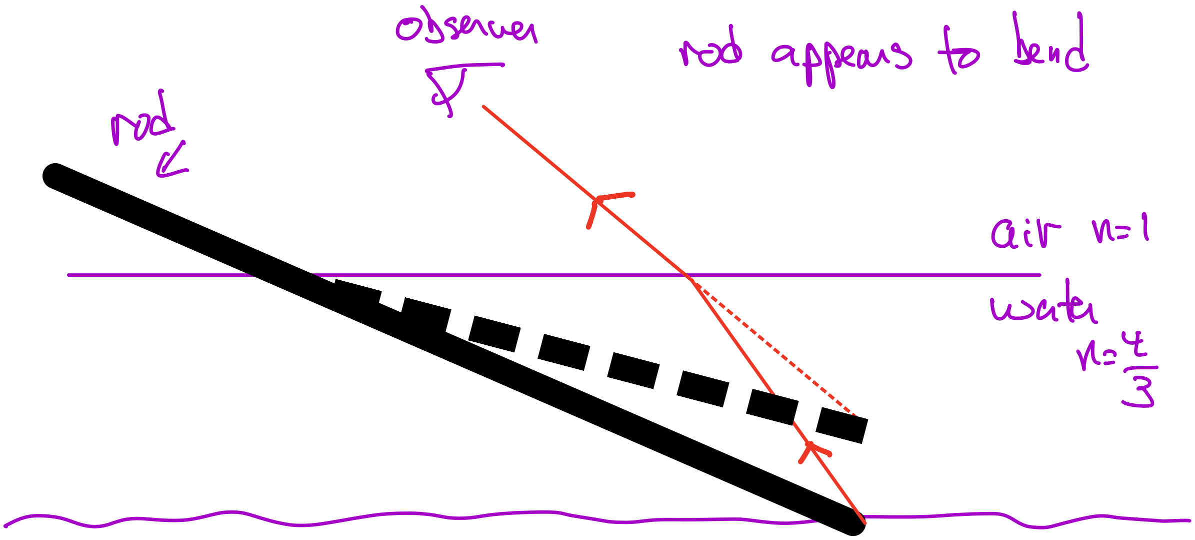
$\cos\theta_2 = \frac{w}{\ell} \Rightarrow \ell = \frac{w}{\cos\theta_2} = \frac{1m}{\cos(28.1°)}$

$= 1.13m$

$\theta_2 + \theta' = \theta_1 = 45° \Rightarrow \theta' = 45 - 28.1 = 16.9°$

then $D = \ell \sin\theta' = 1.13m \cdot \sin(16.9°) = 0.33m$

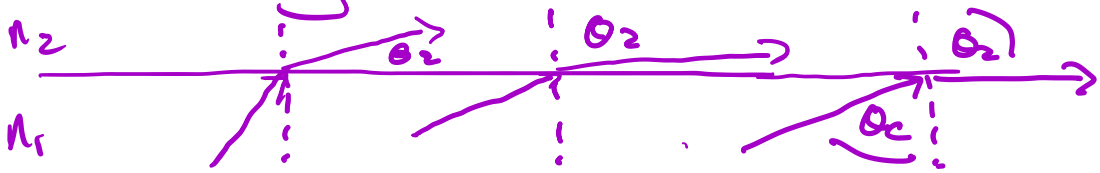
"Optical illusion" Δ objects underwater



Total internal reflection

going from region 1 to 2 where $n_1 > n_2$
(n is decreasing!)

$\Rightarrow$ bent away from normal

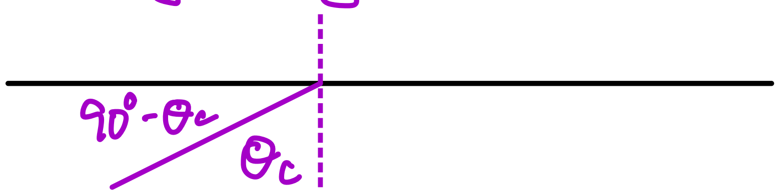


at some incident "critical angle" θ_c, $\theta_2 = 90°$

$$n_1 \sin\theta_c = n_2 \sin 90° = n_2$$

any angle $\theta_1 > \theta_c$ will not be transmitted out of region 1 $\Rightarrow$ total internal reflection
$\Rightarrow$ remember θ is with respect to normal

so θ_c is smallest incident angle for transmission
$\Rightarrow 90° - \theta_c$ is largest angle for total internal reflection



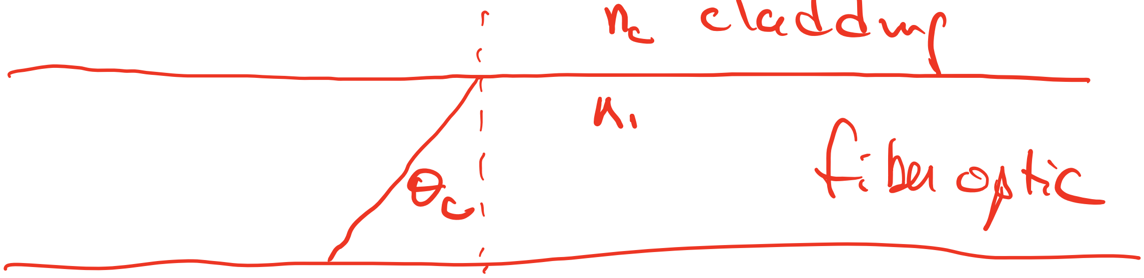
ex: water $n = 1.3$, air $= 1.0$

$$1.3 \sin\theta_c = 1 \Rightarrow \sin\theta_c = \frac{1}{1.3} \Rightarrow \theta_c = 50.3°$$

remember θ_1 is angle wrt normal!

so any $\theta_1 > 50.3°$ will be internally reflected

Fiber Optics



want to make critical angle small so that more light rays stay in fiber

$$\sin\theta_c = \frac{n_{cladding}}{n_{core}}$$

example:

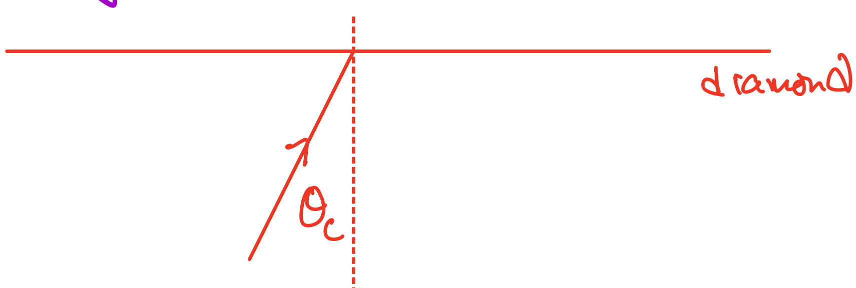
glass: $n_{core} = 1.5$

$$\sin\theta_c = \frac{1}{1.5} = 0.67 \quad \theta_c = 41.8^0$$

diamond: $n_{core} = 2.4$

$$\sin\theta_c = \frac{1}{2.4} = 0.42 \quad \theta_c = 24.6^0$$

that means any light ray exiting at $\theta > \theta_c$ will stay inside



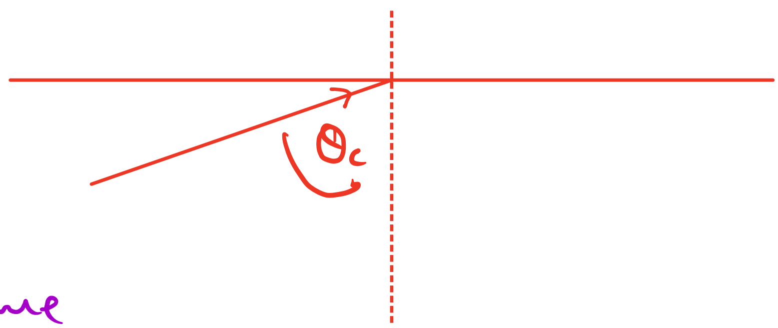
Fiber optics use cladding & cores that have similar index of refraction

ex: core is silica doped w/ germanium $n = 1.465$

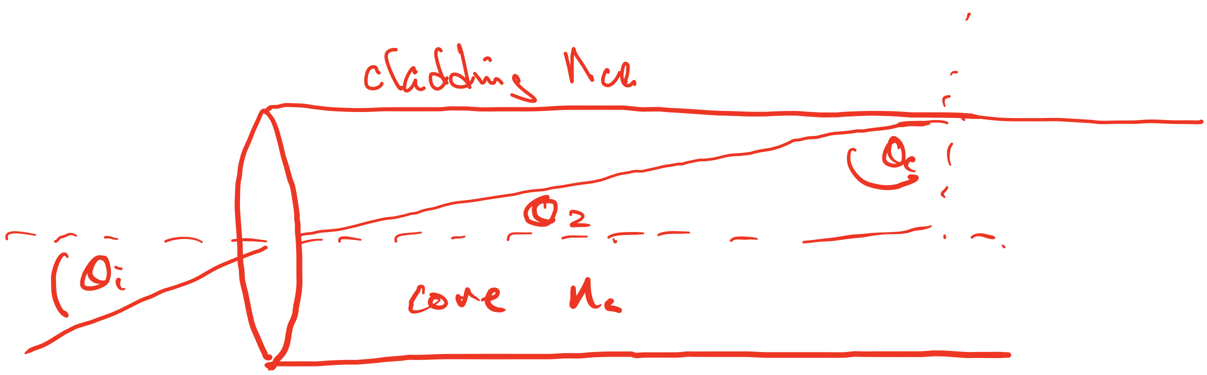
cladding is pure silica $n = 1.450$

$$\sin\theta_c = \frac{n_{cladding}}{n_{core}} = \frac{1.45}{1.465} = 0.99$$

$$\theta_c = 81.79$$



Aperture



θ_i is the entering angle in air, $n_i = 1$

so $\sin\theta_i = n_c \sin\theta_2$

$\theta_2 + \theta_c = 90°$ so $n_c \sin\theta_2 = n_c \sin(90° - \theta_c)$

$= n_c \cos\theta_c$ for total internal reflection (TIR)

$\sin\theta_i = n_c \cos\theta_c \Rightarrow \cos\theta_c = \frac{\sin\theta_i}{n_c}$

$\sin\theta_c = \frac{n_{cl}}{n_c}$ TIR

$\frac{\sin\theta_i}{n_c} = \cos\theta_c$

$\cos^2\theta_c + \sin^2\theta_c = 1 = \frac{\sin^2\theta_i}{n_c^2} + \frac{n_{cl}^2}{n_c^2}$ solve for θ_i

0.21

$\sin^2\theta_i = n_c^2 - n_{cl}^2$

or $\boxed{\sin\theta_i = \sqrt{n_c^2 - n_{cl}^2}}$ aperture

for above case, $\sin\theta_i = \sqrt{1.465^2 - 1.45^2} = 0.21$

$\theta_i = 12.1°$

this is the "aperture" and tells you what entering angles will keep the light inside the fiber ($\theta > \theta_{ap}$ won't stay in)

$\Rightarrow$ it is a very shallow angle to minimize light bouncing so that the pulse doesn't "spread"

200 2/24 400 2/24

Dispersion

"white light" is made up of all colors

some materials have index of refraction that is dependent on wavelength ⇒ $n(\lambda)$

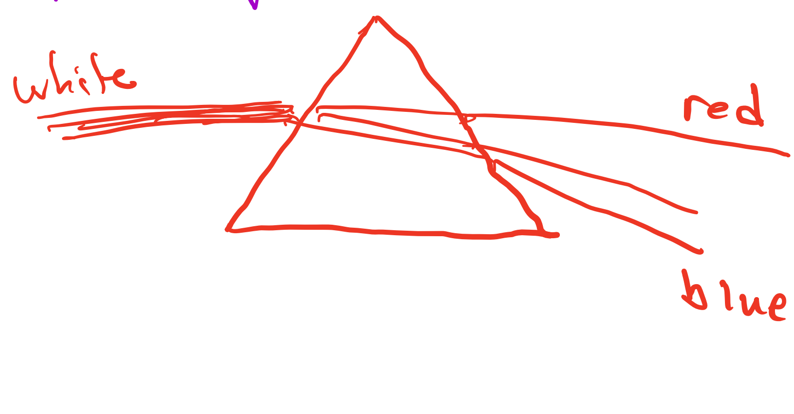
⇒ this will disperse different colors during refraction

usually $n(\lambda)$ is bigger for smaller (bluer)

This is very useful in separating wavelengths

Quartz, silicate flint glass, etc. is dispersive

Prisms disperse light



used in astronomy to detect wavelengths

Big Bang: all galaxies are moving apart

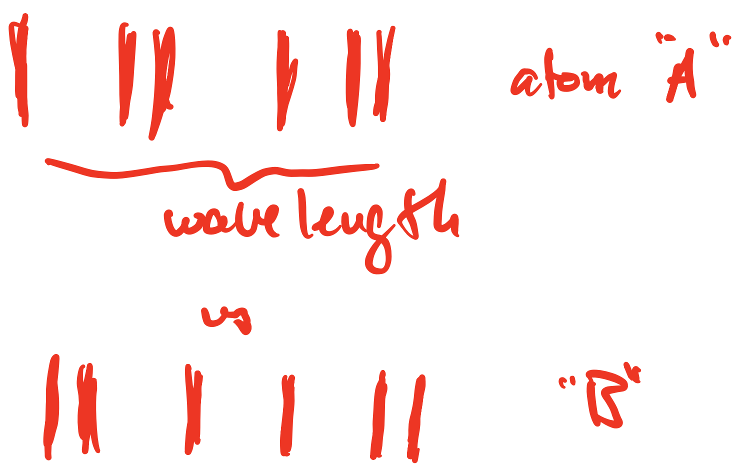
⇒ length scales are increasing !

(balloon analogy)

⇒ the further away a galaxy is, the faster it's receding

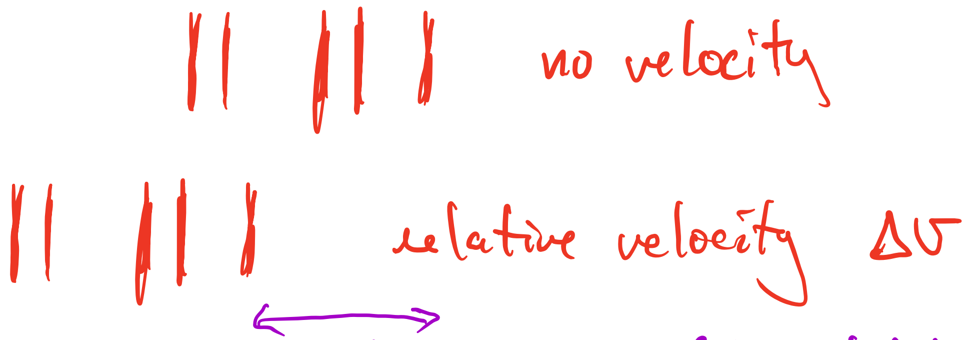
Atomic spectra: atoms excite & deexcite and emit photons

spectrum of light emitted identifies the atom



astronomers use prisms to disperse light so they can see the pattern

the wavelengths will be doppler shifted due to the relative motion

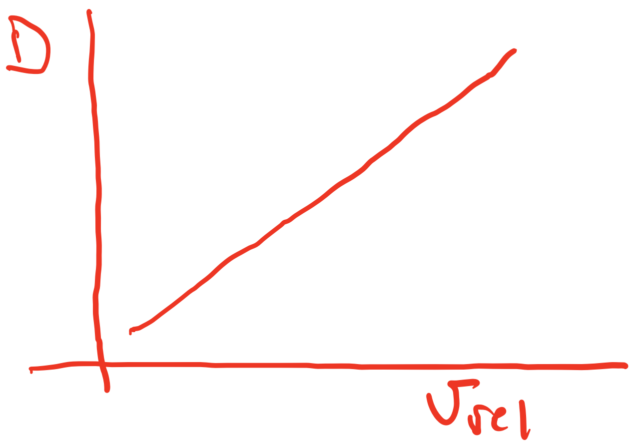


red shift related to v_{rel}

$v = \lambda f$

$\Delta v = f \Delta\lambda$ (close enough, need special relativity)

Hubble's constant – relates distance to v_{rel}



by using prisms & atomic spectroscopy we can map the galaxies in the universe!

Polarization

this concerns the wave nature of light

(from wave equation $\Rightarrow \vec{E} = E_0 \hat{x} \cos(kx - \omega t)$

here the vector nature is constant along $\hat{x}$ direction

waves w/ constant direction are <u>linearly polarized</u>

can make materials that have "preferred" directions, will only let light thru if polarization is parallel to that direction

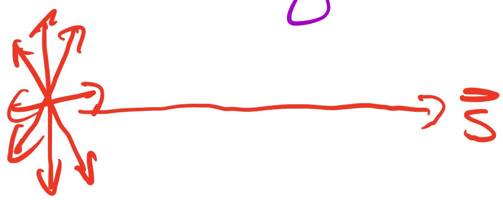
$\Rightarrow$ called polarizing filter, or polarizer

Polarizer direction of EM wave is usually the direction of $\vec{E}$ field, not $\vec{B}$

For polarizer filter, want direction to match to get transmission

Polarizer filters can pick out direction of light polarization unpolarized light

superposition of lots of light waves w/random polarization



Polarizer filter will only allow light polarized along direction of polarization of filter to pass

Sunglasses filter intensity so light is not too bright

=> if I_0 is initial intensity of unpolarized light then transmitted intensity $I_t = I_0/2$ after filter

What happens when light hits a material?

=> light = EM waves, w/ E fields
these fields accelerate atoms

Reflected light is the reemission away from the surface!

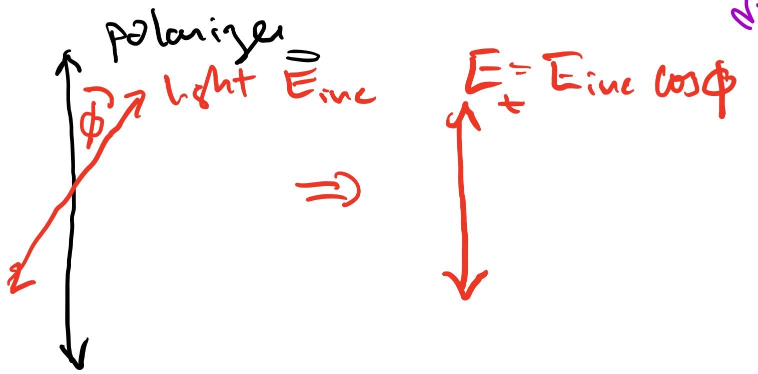
Refracted light is reemission inside the material

The treatment of reflection & refraction at the atomic level is very complex!

The thing to understand:

- light consists of oscillating E-fields that accelerate atoms
- accelerating atoms radiate light

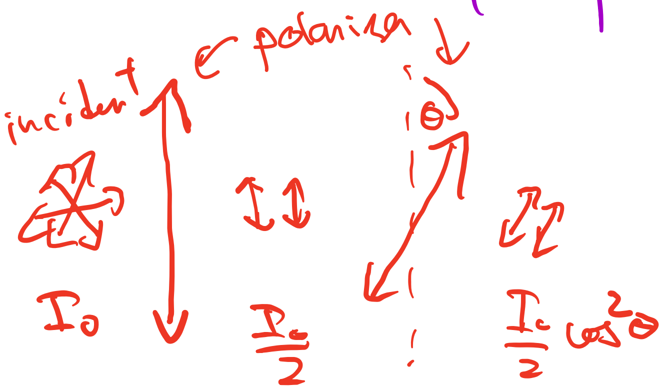
Polarized intensity - only light oscillating along polarizer direction is transmitted



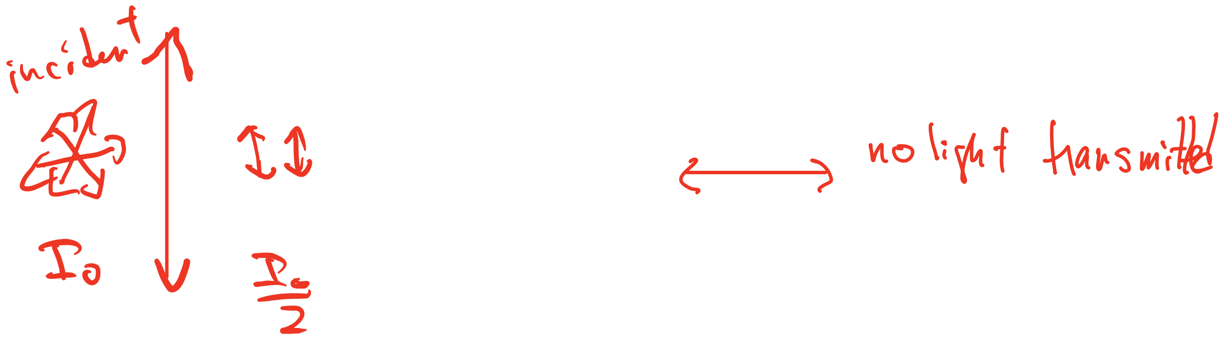
Intensity $I \propto E^2$ so transmitted intensity I_{trans}

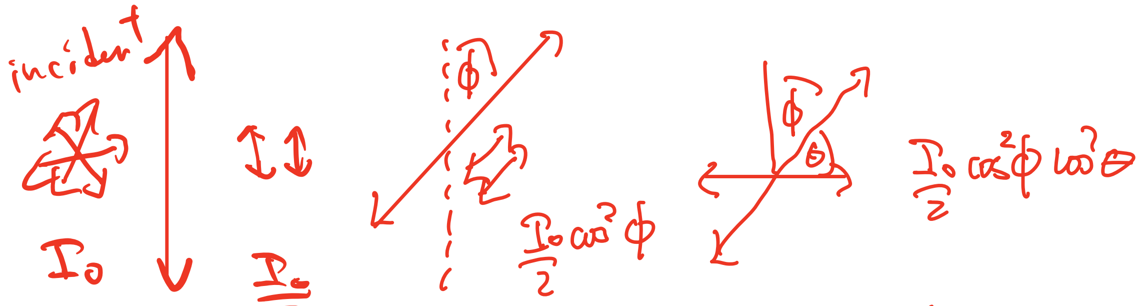
$$I_{trans} = I_{incident} \cdot \cos^2\phi$$

do demo w/ 3 polaroid filters



demo w/ 3 polarizers

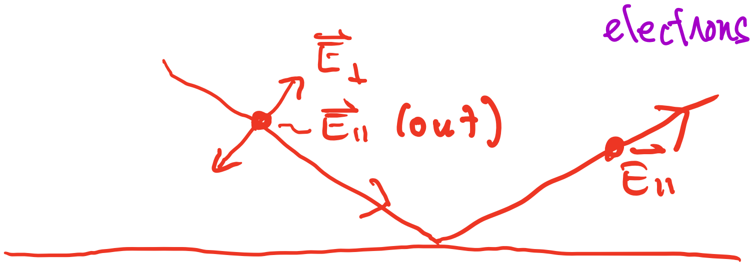




$\cos\theta = \sin\phi$

so $I_{trans} = \frac{I_0}{2}\cos^2\phi\sin^2\phi$

Reflection at a surface ⇒ $E_\perp$ won't accelerate electrons ⊥ surface so no $E_\perp$ re-radiating



the incoming wave will have an oscillating E-field:

$$\vec{E} = \vec{E}_{\parallel} + \vec{E}_\perp$$

$\vec{E}_{\parallel}$ is parallel to surface

$\vec{E}_\perp$ is ⊥ to surface

remember: $\vec{E}$ ⊥ direction of propagation

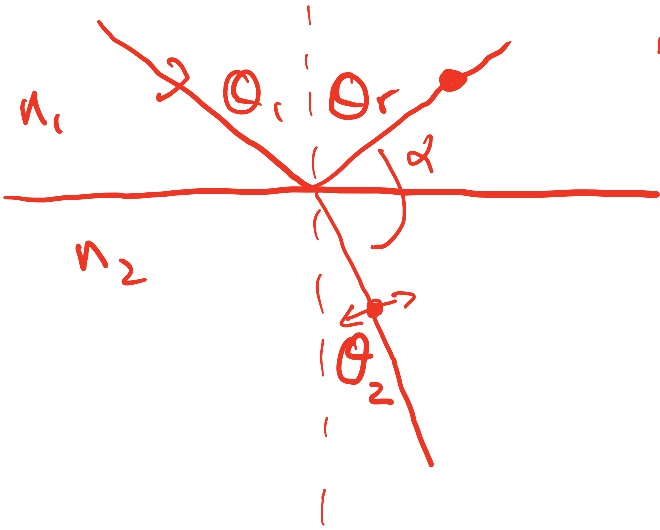
atoms on the surface will tend to move along surface and not <u>into</u> surface

most of $\vec{E}_{\parallel}$ component causes acceleration

" " " " is reflected

⇒ Reflected light is partially polarized parallel to surface of reflection incident unpolarized light at θ_1

(slide 28)



reflected light is polarized parallel to ground

refracted light has both ∥ & ⊥ polarizations

⇒ what if $\theta_2 + \theta_r = 90°$? then $\alpha = 90°$

conservation of electric field: transmitted light is all ⊥ polarized so reflected light is completely polarized ∥ to surface

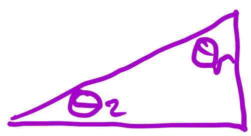
this condition:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \theta_1 = \theta_r$$

$$= n_2 \cos\theta_r \qquad \theta_2 = 90 - \theta_r$$

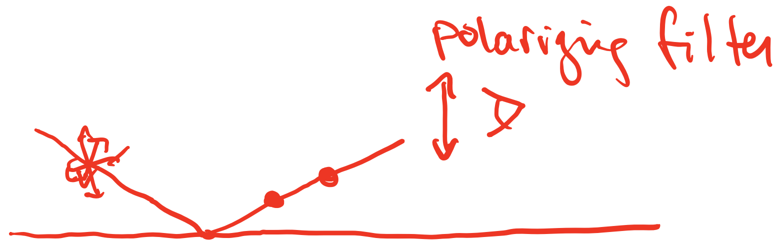
$$= n_2 \cos\theta_1$$

so $\boxed{\tan\theta_1 = \frac{n_2}{n_1}}$



$\sin\theta_2 = \cos\theta_r$

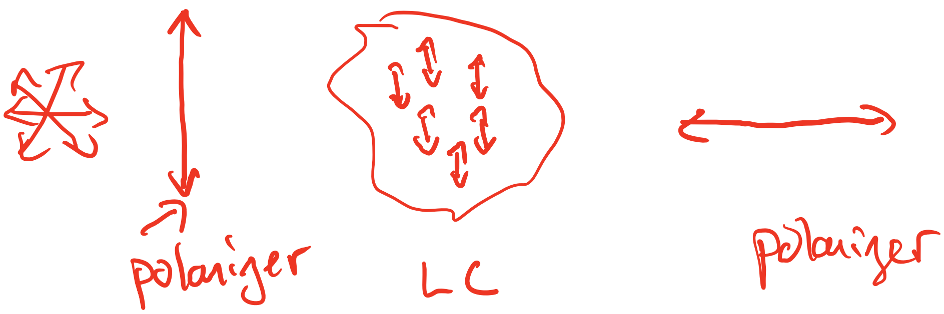
polaroid sunglasses:


polarizing filter

Liquid crystals

liquids w/ crystal domains like iron

- 1st polarizer will only allow 1 direction of polarization to pass
- LC will rotate light so it gets thru 2nd filter
- apply voltage to LC & crystals line up



2nd filter blocks all light

voltages can control whether pixel is on/off

LCD: has array of pixels + back light
computer controls which pixel is on/off

color is implemented using color filters

=> contrast is important in LCD's
- can make LC layer thicker to increase contrast
- but switching LC direction is not instant

so thickness × time ~ constant

faster LCD's mean less contrast
more contrast means can't refresh as fast

200 2/26

400 2/26